#### Sample Problem 5.01 One- and two-dimensional forces, puck

Here are examples of how to use Newton's second law for a puck when one or two forces act on it. Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is m = 0.20 kg. Forces  $\vec{F}_1$  and  $\vec{F}_2$  are directed along the axis and have magnitudes  $F_1 = 4.0$  N and  $F_2 = 2.0$  N. Force  $\vec{F}_3$  is directed at angle  $\theta = 30^\circ$  and has magnitude  $F_3 = 1.0$  N. In each situation, what is the acceleration of the puck?

#### **KEY IDEA**

In each situation we can relate the acceleration  $\vec{a}$  to the net force  $\vec{F}_{net}$  acting on the puck with Newton's second law,  $\vec{F}_{net} = m \vec{a}$ . However, because the motion is along only the *x* axis, we can simplify each situation by writing the second law for *x* components only:

$$F_{\text{net},x} = ma_x. \tag{5-4}$$

The free-body diagrams for the three situations are also given in Fig. 5-3, with the puck represented by a dot.

*Situation A:* For Fig. 5-3*b*, where only one horizontal force acts, Eq. 5-4 gives us

$$F_1 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1}{m} = \frac{4.0 \text{ N}}{0.20 \text{ kg}} = 20 \text{ m/s}^2.$$
 (Answer)

The positive answer indicates that the acceleration is in the positive direction of the x axis.

**Situation B:** In Fig. 5-3*d*, two horizontal forces act on the puck,  $\vec{F}_1$  in the positive direction of *x* and  $\vec{F}_2$  in the negative direction. Now Eq. 5-4 gives us

$$F_1 - F_2 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0 \text{ N} - 2.0 \text{ N}}{0.20 \text{ kg}} = 10 \text{ m/s}^2.$$

(Answer)

Thus, the net force accelerates the puck in the positive direction of the *x* axis.

**Situation C:** In Fig. 5-3*f*, force  $\vec{F}_3$  is not directed along the direction of the puck's acceleration; only *x* component  $F_{3,x}$  is. (Force  $\vec{F}_3$  is two-dimensional but the motion is only



**Figure 5-3** In three situations, forces act on a puck that moves along an *x* axis. Free-body diagrams are also shown.

one-dimensional.) Thus, we write Eq. 5-4 as

$$F_{3,x} - F_2 = ma_x. (5-5)$$

From the figure, we see that  $F_{3,x} = F_3 \cos \theta$ . Solving for the acceleration and substituting for  $F_{3,x}$  yield

$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos \theta - F_2}{m}$$
$$= \frac{(1.0 \text{ N})(\cos 30^\circ) - 2.0 \text{ N}}{0.20 \text{ kg}} = -5.7 \text{ m/s}^2.$$
(Answer)

Thus, the net force accelerates the puck in the negative direction of the x axis.

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acts on a particle, we can find the force as

$$F(x) = -\frac{dU(x)}{dx}.$$
(8-22)

If U(x) is given on a graph, then at any value of x, the force F(x) is the negative of the slope of the curve there and the kinetic energy of the particle is given by

$$K(x) = E_{mec} - U(x),$$
 (8-24)

where  $E_{\text{mec}}$  is the mechanical energy of the system. A **turning point** is a point x at which the particle reverses its motion (there, K = 0). The particle is in **equilibrium** at points where the slope of the U(x) curve is zero (there, F(x) = 0).

**Work Done on a System by an External Force** Work *W* is energy transferred to or from a system by means of an external force acting on the system. When more than one force acts on a system, their *net work* is the transferred energy. When friction is not involved, the work done on the system and the change  $\Delta E_{mec}$  in the mechanical energy of the system are equal:

$$W = \Delta E_{\text{mec}} = \Delta K + \Delta U. \qquad (8-26, 8-25)$$

When a kinetic frictional force acts within the system, then the thermal energy  $E_{\rm th}$  of the system changes. (This energy is associated with the random motion of atoms and molecules in the system.) The work done on the system is then

$$W = \Delta E_{\rm mec} + \Delta E_{\rm th}.$$
 (8-33)



1 In Fig. 8-18, a horizontally moving block can take three frictionless routes, differing only in elevation, to reach the dashed finish line. Rank the routes according to (a) the speed of the block at the finish line and (b) the travel time of the block to the finish line, greatest first.



Figure 8-18 Question 1.

**2** Figure 8-19 gives the potential energy function of a particle. (a) Rank regions AB, BC, CD, and DE according to the



The change  $\Delta E_{\text{th}}$  is related to the magnitude  $f_k$  of the frictional force and the magnitude d of the displacement caused by the external force by

$$\Delta E_{\rm th} = f_k d. \tag{8-31}$$

**Conservation of Energy** The total energy E of a system (the sum of its mechanical energy and its internal energies, including thermal energy) can change only by amounts of energy that are transferred to or from the system. This experimental fact is known as the **law of conservation of energy.** If work W is done on the system, then

$$W = \Delta E = \Delta E_{\rm mec} + \Delta E_{\rm th} + \Delta E_{\rm int}.$$
 (8-35)

If the system is isolated (W = 0), this gives

$$\Delta E_{\rm mec} + \Delta E_{\rm th} + \Delta E_{\rm int} = 0 \tag{8-36}$$

nd 
$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}},$$
 (8-37)

where the subscripts 1 and 2 refer to two different instants.

**Power** The **power** due to a force is the *rate* at which that force transfers energy. If an amount of energy  $\Delta E$  is transferred by a force in an amount of time  $\Delta t$ , the **average power** of the force is

$$P_{\rm avg} = \frac{\Delta E}{\Delta t}.$$
 (8-40)

The instantaneous power due to a force is

$$P = \frac{dE}{dt}.$$
(8-41)

magnitude of the force on the particle, greatest first. What value must the mechanical energy  $E_{mec}$  of the particle not exceed if the particle is to be (b) trapped in the potential well at the left, (c) trapped in the potential well at the right, and (d) able to move between the two potential wells but not to the right of point *H*? For the situation of (d), in which of regions *BC*, *DE*, and *FG* will the particle have (e) the greatest kinetic energy and (f) the least speed?

**3** Figure 8-20 shows one direct path and four indirect paths from point *i* to point *f*. Along the direct path and three of the indirect paths, only a conservative force  $F_c$  acts on a certain object. Along the fourth indirect path, both  $F_c$  and a nonconservative force  $F_{nc}$  act on the object. The change  $\Delta E_{mec}$  in the object's



Figure 8-20 Question 3.

mechanical energy (in joules) in going from *i* to *f* is indicated along each straight-line segment of the indirect paths. What is  $\Delta E_{mec}$  (a) from *i* to *f* along the direct path and (b) due to  $F_{nc}$  along the one path where it acts?

**4** In Fig. 8-21, a small, initially stationary block is released on a frictionless ramp at a height of 3.0 m. Hill heights along the ramp are as shown in the figure. The hills have identical circular tops, and the block does not fly off any hill. (a) Which hill is the first the block cannot cross? (b) What does the block do after failing to cross that hill? Of the hills that the block can cross, on which

acceleration  $a_{\text{com},x}$  down the ramp. We do this by using Newton's second law in both its linear version ( $F_{\text{net}} = Ma$ ) and its angular version ( $\tau_{\text{net}} = Ia$ ).

We start by drawing the forces on the body as shown in Fig. 11-8:

- 1. The gravitational force  $\vec{F}_g$  on the body is directed downward. The tail of the vector is placed at the center of mass of the body. The component along the ramp is  $F_g \sin \theta$ , which is equal to  $Mg \sin \theta$ .
- 2. A normal force  $\vec{F}_N$  is perpendicular to the ramp. It acts at the point of contact *P*, but in Fig. 11-8 the vector has been shifted along its direction until its tail is at the body's center of mass.
- **3.** A static frictional force  $\overline{f}_s$  acts at the point of contact *P* and is directed up the ramp. (Do you see why? If the body were to slide at *P*, it would slide *down* the ramp. Thus, the frictional force opposing the sliding must be *up* the ramp.)

We can write Newton's second law for components along the x axis in Fig. 11-8  $(F_{\text{net},x} = ma_x)$  as

$$f_s - Mg\sin\theta = Ma_{\text{com},x}.$$
 (11-7)

This equation contains two unknowns,  $f_s$  and  $a_{com,x}$ . (We should *not* assume that  $f_s$  is at its maximum value  $f_{s,max}$ . All we know is that the value of  $f_s$  is just right for the body to roll smoothly down the ramp, without sliding.)

We now wish to apply Newton's second law in angular form to the body's rotation about its center of mass. First, we shall use Eq. 10-41 ( $\tau = r_{\perp}F$ ) to write the torques on the body about that point. The frictional force  $\vec{f}_s$  has moment arm R and thus produces a torque  $Rf_s$ , which is positive because it tends to rotate the body counterclockwise in Fig. 11-8. Forces  $\vec{F}_g$  and  $\vec{F}_N$  have zero moment arms about the center of mass and thus produce zero torques. So we can write the angular form of Newton's second law ( $\tau_{net} = I\alpha$ ) about an axis through the body's center of mass as

$$Rf_s = I_{\rm com}\alpha. \tag{11-8}$$

This equation contains two unknowns,  $f_s$  and  $\alpha$ .

Because the body is rolling smoothly, we can use Eq. 11-6 ( $a_{com} = \alpha R$ ) to relate the unknowns  $a_{com,x}$  and  $\alpha$ . But we must be cautious because here  $a_{com,x}$  is negative (in the negative direction of the x axis) and  $\alpha$  is positive (counterclockwise). Thus we substitute  $-a_{com,x}/R$  for  $\alpha$  in Eq. 11-8. Then, solving for  $f_s$ , we obtain

$$f_s = -I_{\rm com} \frac{a_{\rm com,x}}{R^2}.$$
 (11-9)

Substituting the right side of Eq. 11-9 for  $f_s$  in Eq. 11-7, we then find

$$a_{\text{com},x} = -\frac{g\sin\theta}{1 + I_{\text{com}}/MR^2}.$$
 (11-10)

We can use this equation to find the linear acceleration  $a_{\text{com},x}$  of any body rolling along an incline of angle  $\theta$  with the horizontal.

Note that the pull by the gravitational force causes the body to come down the ramp, but it is the frictional force that causes the body to rotate and thus roll. If you eliminate the friction (by, say, making the ramp slick with ice or grease) or arrange for  $Mg \sin \theta$  to exceed  $f_{s,max}$ , then you eliminate the smooth rolling and the body slides down the ramp.

## Checkpoint 2

Disks *A* and *B* are identical and roll across a floor with equal speeds. Then disk *A* rolls up an incline, reaching a maximum height *h*, and disk *B* moves up an incline that is identical except that it is frictionless. Is the maximum height reached by disk *B* greater than, less than, or equal to h?



**Figure 14-16** Fluid flows at a constant speed v through a tube. (a) At time t, fluid element e is about to pass the dashed line. (b) At time  $t + \Delta t$ , element e is a distance  $\Delta x = v \Delta t$  from the dashed line.



**Figure 14-17** A tube of flow is defined by the streamlines that form the boundary of the tube. The volume flow rate must be the same for all cross sections of the tube of flow.

We can use this common volume  $\Delta V$  to relate the speeds and areas. To do so, we first consider Fig. 14-16, which shows a side view of a tube of *uniform* cross-sectional area A. In Fig. 14-16a, a fluid element e is about to pass through the dashed line drawn across the tube width. The element's speed is v, so during a time interval  $\Delta t$ , the element moves along the tube a distance  $\Delta x = v \Delta t$ . The volume  $\Delta V$  of fluid that has passed through the dashed line in that time interval  $\Delta t$  is

$$\Delta V = A \ \Delta x = Av \ \Delta t. \tag{14-22}$$

Applying Eq. 14-22 to both the left and right ends of the tube segment in Fig. 14-15, we have

 $\Delta V = A_1 v_1 \ \Delta t = A_2 v_2 \ \Delta t$ 

or

 $A_1v_1 = A_2v_2$  (equation of continuity). (14-23)

This relation between speed and cross-sectional area is called the **equation of continuity** for the flow of an ideal fluid. It tells us that the flow speed increases when we decrease the cross-sectional area through which the fluid flows.

Equation 14-23 applies not only to an actual tube but also to any so-called *tube of flow*, or imaginary tube whose boundary consists of streamlines. Such a tube acts like a real tube because no fluid element can cross a streamline; thus, all the fluid within a tube of flow must remain within its boundary. Figure 14-17 shows a tube of flow in which the cross-sectional area increases from area  $A_1$  to area  $A_2$  along the flow direction. From Eq. 14-23 we know that, with the increase in area, the speed must decrease, as is indicated by the greater spacing between streamlines at the right in Fig. 14-17. Similarly, you can see that in Fig. 14-13 the speed of the flow is greatest just above and just below the cylinder.

We can rewrite Eq. 14-23 as

$$R_V = Av = a \text{ constant}$$
 (volume flow rate, equation of continuity), (14-24)

in which  $R_V$  is the **volume flow rate** of the fluid (volume past a given point per unit time). Its SI unit is the cubic meter per second (m<sup>3</sup>/s). If the density  $\rho$  of the fluid is uniform, we can multiply Eq. 14-24 by that density to get the **mass flow rate**  $R_m$  (mass per unit time):

$$R_m = \rho R_V = \rho A v = a \text{ constant}$$
 (mass flow rate). (14-25)

The SI unit of mass flow rate is the kilogram per second (kg/s). Equation 14-25 says that the mass that flows into the tube segment of Fig. 14-15 each second must be equal to the mass that flows out of that segment each second.



The figure shows a pipe and gives the volume flow rate (in cm<sup>3</sup>/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section?





**Figure 17-20** The wavefronts of Fig. 17-19, assumed planar, (*a*) reach and (*b*) pass a stationary detector *D*; they move a distance *vt* to the right in time *t*.



**Figure 17-21** Wavefronts traveling to the right (*a*) reach and (*b*) pass detector *D*, which moves in the opposite direction. In time *t*, the wavefronts move a distance vt to the right and *D* moves a distance  $v_Dt$  to the left.

**Figure 17-19** A stationary source of sound *S* emits spherical wavefronts, shown one wavelength apart, that expand outward at speed *v*. A sound detector *D*, represented by an ear, moves with velocity  $\vec{v}_D$  toward the source. The detector senses a higher frequency because of its motion.

### **Detector Moving, Source Stationary**

In Fig. 17-19, a detector D (represented by an ear) is moving at speed  $v_D$  toward a stationary source S that emits spherical wavefronts, of wavelength  $\lambda$  and frequency f, moving at the speed v of sound in air. The wavefronts are drawn one wavelength apart. The frequency detected by detector D is the rate at which Dintercepts wavefronts (or individual wavelengths). If D were stationary, that rate would be f, but since D is moving into the wavefronts, the rate of interception is greater, and thus the detected frequency f' is greater than f.

Let us for the moment consider the situation in which *D* is stationary (Fig. 17-20). In time *t*, the wavefronts move to the right a distance *vt*. The number of wavelengths in that distance *vt* is the number of wavelengths intercepted by *D* in time *t*, and that number is  $vt/\lambda$ . The rate at which *D* intercepts wavelengths, which is the frequency *f* detected by *D*, is

$$f = \frac{vt/\lambda}{t} = \frac{v}{\lambda}.$$
 (17-48)

In this situation, with D stationary, there is no Doppler effect – the frequency detected by D is the frequency emitted by S.

Now let us again consider the situation in which D moves in the direction opposite the wavefront velocity (Fig. 17-21). In time t, the wavefronts move to the right a distance vt as previously, but now D moves to the left a distance  $v_D t$ . Thus, in this time t, the distance moved by the wavefronts relative to D is  $vt + v_D t$ . The number of wavelengths in this relative distance  $vt + v_D t$  is the number of wavelengths intercepted by D in time t and is  $(vt + v_D t)/\lambda$ . The rate at which D intercepts wavelengths in this situation is the frequency f', given by

$$f' = \frac{(vt + v_D t)/\lambda}{t} = \frac{v + v_D}{\lambda}.$$
(17-49)

From Eq. 17-48, we have  $\lambda = v/f$ . Then Eq. 17-49 becomes

$$f' = \frac{v + v_D}{v/f} = f \frac{v + v_D}{v}.$$
 (17-50)

Note that in Eq. 17-50, f' > f unless  $v_D = 0$  (the detector is stationary).

Similarly, we can find the frequency detected by D if D moves away from the source. In this situation, the wavefronts move a distance  $vt - v_D t$  relative to D in time t, and f' is given by

$$f' = f \frac{v - v_D}{v}.\tag{17-51}$$

In Eq. 17-51, f' < f unless  $v_D = 0$ . We can summarize Eqs. 17-50 and 17-51 with

$$f' = f \frac{v \pm v_D}{v}$$
 (detector moving, source stationary). (17-52)

Shift up: The detector moves *toward* the source.





**Figure 20-18** For a *large* number of molecules in a box, a plot of the number of microstates that require various percentages of the molecules to be in the left half of the box. Nearly all the microstates correspond to an approximately equal sharing of the molecules between the two halves of the box; those microstates form the *central configuration peak* on the plot. For  $N \approx 10^{22}$ , the central configuration peak is much too narrow to be drawn on this plot.

You should verify the multiplicities for all the configurations in Table 20-1. The basic assumption of statistical mechanics is that

All microstates are equally probable.

In other words, if we were to take a great many snapshots of the six molecules as they jostle around in the box of Fig. 20-17 and then count the number of times each microstate occurred, we would find that all 64 microstates would occur equally often. Thus the system will spend, on average, the same amount of time in each of the 64 microstates.

Because all microstates are equally probable but different configurations have different numbers of microstates, the configurations are *not* all equally probable. In Table 20-1 configuration IV, with 20 microstates, is the *most probable configuration*, with a probability of 20/64 = 0.313. This result means that the system is in configuration IV 31.3% of the time. Configurations I and VII, in which all the molecules are in one half of the box, are the least probable, each with a probability of 1/64 = 0.016 or 1.6%. It is not surprising that the most probable configuration is the one in which the molecules are evenly divided between the two halves of the box, because that is what we expect at thermal equilibrium. However, it *is* surprising that there is *any* probability, however small, of finding all six molecules clustered in half of the box, with the other half empty.

For large values of *N* there are extremely large numbers of microstates, but nearly all the microstates belong to the configuration in which the molecules are divided equally between the two halves of the box, as Fig. 20-18 indicates. Even though the measured temperature and pressure of the gas remain constant, the gas is churning away endlessly as its molecules "visit" all probable microstates with equal probability. However, because so few microstates lie outside the very narrow central configuration peak of Fig. 20-18, we might as well assume that the gas molecules are always divided equally between the two halves of the box. As we shall see, this is the configuration with the greatest entropy.

#### Sample Problem 20.05 Microstates and multiplicity

Suppose that there are 100 indistinguishable molecules in the box of Fig. 20-17. How many microstates are associated with the configuration  $n_1 = 50$  and  $n_2 = 50$ , and with the configuration  $n_1 = 100$  and  $n_2 = 0$ ? Interpret the results in terms of the relative probabilities of the two configurations.

#### **KEY IDEA**

The multiplicity *W* of a configuration of indistinguishable molecules in a closed box is the number of independent microstates with that configuration, as given by Eq. 20-20.

**Calculations:** Thus, for the  $(n_1, n_2)$  configuration (50, 50),

$$W = \frac{N!}{n_1! n_2!} = \frac{100!}{50! 50!}$$
  
=  $\frac{9.33 \times 10^{157}}{(3.04 \times 10^{64})(3.04 \times 10^{64})}$   
=  $1.01 \times 10^{29}$ . (Answer)

Similarly, for the configuration (100, 0), we have

$$W = \frac{N!}{n_1! n_2!} = \frac{100!}{100! 0!} = \frac{1}{0!} = \frac{1}{1} = 1.$$
 (Answer)

**The meaning:** Thus, a 50–50 distribution is more likely than a 100–0 distribution by the enormous factor of about  $1 \times 10^{29}$ . If you could count, at one per nanosecond, the number of microstates that correspond to the 50–50 distribution, it would take you about  $3 \times 10^{12}$  years, which is about 200 times longer than the age of the universe. Keep in mind that the 100 molecules used in this sample problem is a very small number. Imagine what these calculated probabilities would be like for a mole of molecules, say about  $N = 10^{24}$ . Thus, you need never worry about suddenly finding all the air molecules clustering in one corner of your room, with you gasping for air in another corner. So, you can breathe easy because of the physics of entropy.

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Since the charge on the rod is positive and we have taken V = 0 at infinity, we know from Module 24-3 that dV in Eq. 24-34 must be positive.

We now find the total potential V produced by the rod at point P by integrating Eq. 24-34 along the length of the rod, from x = 0 to x = L (Figs. 24-15d and e), using integral 17 in Appendix E. We find

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx$$
$$= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$
$$= \frac{\lambda}{4\pi\varepsilon_0} \left[ \ln\left(x + (x^2 + d^2)^{1/2}\right) \right]_0^L$$
$$= \frac{\lambda}{4\pi\varepsilon_0} \left[ \ln\left(L + (L^2 + d^2)^{1/2}\right) - \ln d \right].$$

We can simplify this result by using the general relation  $\ln A - \ln B = \ln(A/B)$ . We then find

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$
 (24-35)

Because V is the sum of positive values of dV, it too is positive, consistent with the logarithm being positive for an argument greater than 1.

#### **Charged Disk**

In Module 22-5, we calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius R that has a uniform charge density  $\sigma$  on one surface. Here we derive an expression for V(z), the electric potential at any point on the central axis. Because we have a circular distribution of charge on the disk, we could start with a differential element that occupies angle  $d\theta$  and radial distance dr. We would then need to set up a two-dimensional integration. However, let's do something easier.

In Fig. 24-16, consider a differential element consisting of a flat ring of radius R' and radial width dR'. Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

in which  $(2\pi R')(dR')$  is the upper surface area of the ring. All parts of this charged element are the same distance *r* from point *P* on the disk's axis. With the aid of Fig. 24-16, we can use Eq. 24-31 to write the contribution of this ring to the electric potential at *P* as

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + {R'}^2}}.$$
 (24-36)

We find the net potential at *P* by adding (via integration) the contributions of all the rings from R' = 0 to R' = R:

$$V = \int dV = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{R' \, dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0} \left(\sqrt{z^2 + R^2} - z\right). \tag{24-37}$$

Note that the variable in the second integral of Eq. 24-37 is R' and not z, which remains constant while the integration over the surface of the disk is carried out. (Note also that, in evaluating the integral, we have assumed that  $z \ge 0$ .)



**Figure 24-16** A plastic disk of radius R, charged on its top surface to a uniform surface charge density  $\sigma$ . We wish to find the potential V at point P on the central axis of the disk.

ideal battery has emf  $\mathscr{E} = 20.0$  V. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time t = 0. What is the current in resistor 2 at t = 4.00 ms?

••66 Figure 27-67 displays two circuits with a charged capacitor that is to be discharged through a resistor when a switch is closed. In Fig. 27-67*a*,  $R_1 = 20.0 \ \Omega$  and  $C_1 = 5.00 \ \mu\text{F}$ . In Fig. 27-67*b*,  $R_2 = 10.0 \ \Omega$  and  $C_2 = 8.00 \ \mu\text{F}$ . The ratio of the initial charges on the two



Figure 27-67 Problem 66.

capacitors is  $q_{02}/q_{01} = 1.50$ . At time t = 0, both switches are closed. At what time t do the two capacitors have the same charge?

••67 The potential difference between the plates of a leaky (meaning that charge leaks from one plate to the other) 2.0  $\mu$ F capacitor drops to one-fourth its initial value in 2.0 s. What is the equivalent resistance between the capacitor plates?

••68 A  $1.0 \,\mu\text{F}$  capacitor with an initial stored energy of 0.50 J is discharged through a 1.0 M $\Omega$  resistor. (a) What is the initial charge on the capacitor? (b) What is the current through the resistor when the discharge starts? Find an expression that gives, as a function of time *t*, (c) the potential difference  $V_C$  across the capacitor, (d) the potential difference  $V_R$  across the resistor, and (e) the rate at which thermal energy is produced in the resistor.

•••69 • A 3.00 M $\Omega$  resistor and a 1.00  $\mu$ F capacitor are connected in series with an ideal battery of emf  $\mathscr{E} = 4.00$  V. At 1.00 s after the connection is made, what is the rate at which (a) the charge of the capacitor is increasing, (b) energy is being stored in the capacitor, (c) thermal energy is appearing in the resistor, and (d) energy is being delivered by the battery?

#### **Additional Problems**

**70 ••** Each of the six real batteries in Fig. 27-68 has an emf of 20 V and a resistance of 4.0  $\Omega$ . (a) What is the current through the (external) resistance  $R = 4.0 \Omega$ ? (b) What is the potential difference across each battery? (c) What is the power of each battery? (d) At what rate does each battery transfer energy to internal thermal energy?

**71** In Fig. 27-69,  $R_1 = 20.0 \Omega$ ,  $R_2 = 10.0 \Omega$ , and the ideal battery has emf  $\mathscr{E} = 120 \text{ V}$ . What is the current at point *a* if we close (a) only switch S<sub>1</sub>, (b) only switches S<sub>1</sub> and S<sub>2</sub>, and (c) all three switches?

**72** In Fig. 27-70, the ideal battery has emf  $\mathscr{C} = 30.0$  V, and the resistances are  $R_1 = R_2 = 14 \Omega$ ,  $R_3 = R_4 = R_5 = 6.0 \Omega$ ,  $R_6 = 2.0 \Omega$ , and  $R_7 =$ 

1.5  $\Omega$ . What are currents (a)  $i_2$ , (b)  $i_4$ , (c)  $i_1$ , (d)  $i_3$ , and (e)  $i_5$ ?



**Figure 27-70** Problem 72.





Figure 27-69 Problem 71.

**73 SSM** Wires *A* and *B*, having equal lengths of 40.0 m and equal diameters of 2.60 mm, are connected in series. A potential difference of 60.0 V is applied between the ends of the composite wire. The resistances are  $R_A = 0.127 \Omega$  and  $R_B = 0.729 \Omega$ . For wire *A*, what are (a) magnitude *J* of the current density and (b) potential difference *V*? (c) Of what type material is wire *A* made (see Table 26-1)? For wire *B*, what are (d) *J* and (e) *V*? (f) Of what type material is *B* made?

**74** What are the (a) size and (b) direction (up or down) of current *i* in Fig. 27-71, where all resistances are 4.0  $\Omega$  and all batteries are ideal and have an emf of 10 V? (*Hint:* This can be answered using only mental calculation.)





**75** Suppose that, while you are sitting in a chair, charge separation between your clothing and the chair puts you at a potential of 200 V, with the capacitance between you and the chair at 150 pF. When you stand up, the increased separation between your body and the chair decreases the capacitance to 10 pF. (a) What then is the potential of your body? That potential is reduced over time, as the charge on you drains through your body and shoes (you are a capacitor discharging through a resistance). Assume that the resistance along that route is 300 G $\Omega$ . If you touch an electrical component while your potential is greater than 100 V, you could ruin the component. (b) How long must you wait until your potential reaches the safe level of 100 V?

If you wear a conducting wrist strap that is connected to ground, your potential does not increase as much when you stand up; you also discharge more rapidly because the resistance through the grounding connection is much less than through your body and shoes. (c) Suppose that when you stand up, your potential is 1400 V and the chair-to-you capacitance is 10 pF. What resistance in that wrist-strap grounding connection will allow you to discharge to 100 V in 0.30 s, which is less time than you would need to reach for, say, your computer?

**76 ••** In Fig. 27-72, the ideal batteries have emfs  $\mathscr{C}_1 = 20.0 \text{ V}$ ,  $\mathscr{C}_2 = 10.0 \text{ V}$ , and  $\mathscr{C}_3 = 5.00 \text{ V}$ , and the resistances are each 2.00  $\Omega$ . What are the (a) size and (b) direction (left or right) of current  $i_1$ ? (c) Does battery 1 supply or absorb energy, and (d) what is its power? (e) Does battery 2 supply or absorb energy, and

axis scale is set by  $\Phi_s = 4.0 \times 10^{-4} \text{ T} \cdot \text{m}^2$ , and the horizontal axis scale is set by  $i_s = 2.00 \text{ A}$ . If switch S is closed at time t = 0, at what rate di/dt will the current be changing at  $t = 1.5\tau_L$ ?

••57 **(a)** In Fig. 30-65,  $R = 15 \Omega$ , L = 5.0 H, the ideal battery has  $\mathscr{E} = 10$  V, and the fuse in the upper branch is an ideal 3.0 A fuse. It has zero resistance as long as the current through it remains less than 3.0 A. If the current reaches 3.0 A, the fuse "blows" and thereafter has infinite resistance. Switch S is closed



Figure 30-65 Problem 57.

at time t = 0. (a) When does the fuse blow? (*Hint:* Equation 30-41 does not apply. Rethink Eq. 30-39.) (b) Sketch a graph of the current *i* through the inductor as a function of time. Mark the time at which the fuse blows.

••58 Suppose the emf of the battery in the circuit shown in Fig. 30-16 varies with time *t* so that the current is given by i(t) = 3.0 + 5.0t, where *i* is in amperes and *t* is in seconds. Take  $R = 4.0 \Omega$  and L = 6.0 H, and find an expression for the battery emf as a function of *t*. (*Hint:* Apply the loop rule.)

•••59 SSM WWW In Fig. 30-66, after switch S is closed at time t = 0, the emf of the source is automatically adjusted to maintain a constant current *i* through S. (a) Find the current through the inductor as a function of time. (b) At what time is the current through the resistor equal to the current through the inductor?



Figure 30-66 Problem 59.

•••60 A wooden toroidal core with a square cross section has an inner radius of 10 cm and an outer radius of 12 cm. It is wound with one layer of wire (of diameter 1.0 mm and resistance per meter  $0.020 \Omega/m$ ). What are (a) the inductance and (b) the inductive time constant of the resulting toroid? Ignore the thickness of the insulation on the wire.

#### Module 30-7 Energy Stored in a Magnetic Field

•61 SSM A coil is connected in series with a 10.0 k $\Omega$  resistor. An ideal 50.0 V battery is applied across the two devices, and the current reaches a value of 2.00 mA after 5.00 ms. (a) Find the inductance of the coil. (b) How much energy is stored in the coil at this same moment?

•62 A coil with an inductance of 2.0 H and a resistance of 10  $\Omega$  is suddenly connected to an ideal battery with  $\mathcal{E} = 100$  V. At 0.10 s after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?

•63 ILW At t = 0, a battery is connected to a series arrangement of a resistor and an inductor. If the inductive time constant is 37.0 ms, at what time is the rate at which energy is dissipated in the resistor equal to the rate at which energy is stored in the inductor's magnetic field?

•64 At t = 0, a battery is connected to a series arrangement of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.500 its steady-state value?

••65 **w** For the circuit of Fig. 30-16, assume that  $\mathcal{C} = 10.0 \text{ V}$ ,  $R = 6.70 \Omega$ , and L = 5.50 H. The ideal battery is connected at time

t = 0. (a) How much energy is delivered by the battery during the first 2.00 s? (b) How much of this energy is stored in the magnetic field of the inductor? (c) How much of this energy is dissipated in the resistor?

#### Module 30-8 Energy Density of a Magnetic Field

•66 A circular loop of wire 50 mm in radius carries a current of 100 A. Find the (a) magnetic field strength and (b) energy density at the center of the loop.

•67 SSM A solenoid that is 85.0 cm long has a cross-sectional area of  $17.0 \text{ cm}^2$ . There are 950 turns of wire carrying a current of 6.60 A. (a) Calculate the energy density of the magnetic field inside the solenoid. (b) Find the total energy stored in the magnetic field there (neglect end effects).

•68 A toroidal inductor with an inductance of 90.0 mH encloses a volume of  $0.0200 \text{ m}^3$ . If the average energy density in the toroid is 70.0 J/m<sup>3</sup>, what is the current through the inductor?

•69 ILW What must be the magnitude of a uniform electric field if it is to have the same energy density as that possessed by a 0.50 T magnetic field?

••70 💿 Figure 30-67*a* shows, in cross section, two wires that are straight, parallel, and very long. The ratio  $i_1/i_2$  of the current carried by wire 1 to that carried by wire 2 is 1/3. Wire 1 is fixed in place. Wire 2 can be moved along the positive side of the x axis so as to change the magnetic energy density  $u_B$ set up by the two currents at the origin. Figure 30-67b gives  $u_B$  as a function of the position x of wire 2. The curve has an asymptote of  $u_B = 1.96 \text{ nJ/m}^3$  as  $x \to \infty$ , and the horizontal axis scale is set by  $x_s = 60.0$  cm. What is the value of (a)  $i_1$  and (b)  $i_2$ ?



••71 A length of copper wire carries a current of 10 A uniformly distributed through its cross section. Calculate the energy density of (a) the magnetic field and (b) the electric field at the surface of the wire. The wire diameter is 2.5 mm, and its resistance per unit length is  $3.3 \Omega/km$ .

#### Module 30-9 Mutual Induction

•72 Coil 1 has  $L_1 = 25$  mH and  $N_1 = 100$  turns. Coil 2 has  $L_2 = 40$  mH and  $N_2 = 200$  turns. The coils are fixed in place; their mutual inductance *M* is 3.0 mH. A 6.0 mA current in coil 1 is changing at the rate of 4.0 A/s. (a) What magnetic flux  $\Phi_{12}$  links coil 1, and (b) what self-induced emf appears in that coil? (c) What magnetic flux  $\Phi_{21}$  links coil 2, and (d) what mutually induced emf appears in that coil?

•73 SSM Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s, the emf in coil 1 is 25.0 mV. (a) What is their mutual inductance? (b) When coil 2 has no current and coil 1 has a current of 3.60 A, what is the flux linkage in coil 2?

•74 Two solenoids are part of the spark coil of an automobile. When the current in one solenoid falls from 6.0 A to zero in 2.5 ms, an emf of 30 kV is induced in the other solenoid. What is the mutual inductance M of the solenoids?

## Questions

1 If the magnetic field of a light wave oscillates parallel to a y axis and is given by  $B_y = B_m \sin(kz - \omega t)$ , (a) in what direction does the wave travel and (b) parallel to which axis does the associated electric field oscillate?

**2** Suppose we rotate the second sheet in Fig. 33-15*a*, starting with the polarization direction aligned with the *y* axis ( $\theta = 0$ ) and ending with it aligned with the *x* axis ( $\theta = 90^{\circ}$ ).



Figure 33-26 Question 2.

Figure 33-27 Question 3.

Figure 33-29 Question 6.

С

Figure 33-30 Question 7.

Which of the four curves in Fig. 33-26 best shows the intensity of the light through the three-sheet system during this  $90^{\circ}$  rotation?

**3** (a) Figure 33-27 shows light reaching a polarizing sheet whose polarizing direction is parallel to a *y* axis. We shall rotate the sheet  $40^{\circ}$  clockwise about the light's indicated line of travel. During this rotation, does the fraction of the initial light intensity passed by the sheet increase, decrease, or remain the same if the light is (a) initially unpolarized, (b) initially polarized parallel to the *x* axis, and (c) initially polarized parallel to the *y* axis?

**4** Figure 33-28 shows the electric and magnetic fields of an electromagnetic wave at a certain instant. Is the wave traveling into the page or out of the page?

5 In the arrangement of Fig. 33-15a, start

with light that is initially polarized parallel to the x axis, and write the ratio of its final intensity  $I_3$  to its initial intensity  $I_0$  as  $I_3/I_0 = A \cos^n \theta$ . What are A, n, and  $\theta$  if we rotate the polarizing direction of the first sheet (a) 60° counterclockwise and (b) 90° clockwise from what is shown?

**6** In Fig. 33-29, unpolarized light is sent into a system of five polarizing sheets. Their polarizing directions, measured counterclockwise from the positive direction of the y axis, are the following: sheet 1,  $35^{\circ}$ ; sheet 2,  $0^{\circ}$ ; sheet 3,  $0^{\circ}$ ; sheet 4,  $110^{\circ}$ ; sheet 5,  $45^{\circ}$ . Sheet 3 is then rotated  $180^{\circ}$  counterclockwise about the light ray. During that rotation, at what angles (measured counterclockwise from the y axis) is the transmission of light through the system eliminated?

**7** Figure 33-30 shows rays of monochromatic light propagating through three materials *a*, *b*, and *c*. Rank the materials according to the index of refraction, greatest first.

8 Figure 33-31 shows the multiple reflections of a light ray along a glass corridor where the walls are either parallel or perpendicular to

one another. If the angle of incidence at point a is  $30^\circ$ , what are the

angles of reflection of the light ray at points b, c, d, e, and f?

**9** Figure 33-32 shows four long horizontal layers A-D of different materials, with air above and below them. The index of refraction of each material is given. Rays of light are sent into the left end of each layer as shown. In which layer is there the possibility of totally trapping the light in that layer so that, after many reflections, all the light reaches the right end of the layer?

**10** The leftmost block in Fig. 33-33 depicts total internal reflection for light inside a material with an index of refraction  $n_1$  when air is outside the material. A light ray reaching point *A* from anywhere within the shaded



Figure 33-31 Question 8.



Figure 33-32 Question 9.

region at the left (such as the ray shown) fully reflects at that point and ends up in the shaded region at the right. The other blocks show similar situations for two other materials. Rank the indexes of refraction of the three materials, greatest first.



Figure 33-33 Question 10.

**11** Each part of Fig. 33-34 shows light that refracts through an interface between two materials. The incident ray (shown gray in the figure) consists of red and blue light. The approximate index of refraction for visible light is indicated for each material. Which of the three parts show physically possible refraction? (*Hint:* First consider the refraction in general, regardless of the color, and then consider how red and blue light refract differently.)





**12** In Fig. 33-35, light travels from material *a*, through three layers of other materials with surfaces parallel to one another, and then back into another layer of material *a*. The refractions (but not the associated reflections) at the surfaces are shown. Rank the materials according to index of refraction, greatest first. (*Hint:* The parallel arrangement of the surfaces allows comparison.)



Figure 33-35 Question 12.





**Figure 36-23** A simple type of grating spectroscope used to analyze the wavelengths of light emitted by source *S*.

We state without proof that the half-width of any other line depends on its location relative to the central axis and is

$$\Delta \theta_{\rm hw} = \frac{\lambda}{Nd \cos \theta} \quad (\text{half-width of line at } \theta). \tag{36-28}$$

Note that for light of a given wavelength  $\lambda$  and a given ruling separation d, the widths of the lines decrease with an increase in the number N of rulings. Thus, of two diffraction gratings, the grating with the larger value of N is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap.

#### **Grating Spectroscope**

Diffraction gratings are widely used to determine the wavelengths that are emitted by sources of light ranging from lamps to stars. Figure 36-23 shows a simple grating spectroscope in which a grating is used for this purpose. Light from source S is focused by lens  $L_1$  on a vertical slit  $S_1$  placed in the focal plane of lens  $L_2$ . The light emerging from tube C (called a collimator) is a plane wave and is incident perpendicularly on grating G, where it is diffracted into a diffraction pattern, with the m = 0 order diffracted at angle  $\theta = 0$  along the central axis of the grating.

We can view the diffraction pattern that would appear on a viewing screen at any angle  $\theta$  simply by orienting telescope *T* in Fig. 36-23 to that angle. Lens  $L_3$  of the telescope then focuses the light diffracted at angle  $\theta$  (and at slightly smaller and larger angles) onto a focal plane *FF'* within the telescope. When we look through eyepiece *E*, we see a magnified view of this focused image.

By changing the angle  $\theta$  of the telescope, we can examine the entire diffraction pattern. For any order number other than m = 0, the original light is spread out according to wavelength (or color) so that we can determine, with Eq. 36-25, just what wavelengths are being emitted by the source. If the source emits discrete wavelengths, what we see as we rotate the telescope horizontally through the angles corresponding to an order *m* is a vertical line of color for each wavelength, with the shorter-wavelength line at a smaller angle  $\theta$  than the longer-wavelength line.

**Hydrogen.** For example, the light emitted by a hydrogen lamp, which contains hydrogen gas, has four discrete wavelengths in the visible range. If our eyes intercept this light directly, it appears to be white. If, instead, we view it through a grating spectroscope, we can distinguish, in several orders, the lines of the four colors corresponding to these visible wavelengths. (Such lines are called *emission lines.*) Four orders are represented in Fig. 36-24. In the central order (m = 0), the lines corresponding to all four wavelengths are superimposed, giving a single white line at  $\theta = 0$ . The colors are separated in the higher orders.

The third order is not shown in Fig. 36-24 for the sake of clarity; it actually overlaps the second and fourth orders. The fourth-order red line is missing because it is not formed by the grating used here. That is, when we attempt to







**Figure 39-13** A rectangular corral – a two-dimensional version of the infinite potential well of Fig. 39-2 – with widths  $L_x$  and  $L_y$ .

This is a three-dimensional trap with infinite potential walls.

**Figure 39-14** A rectangular box – a three-dimensional version of the infinite potential well of Fig. 39-2 – with widths  $L_x$ ,  $L_y$ , and  $L_z$ .

## **Two- and Three-Dimensional Electron Traps**

In the next module, we shall discuss the hydrogen atom as being a threedimensional finite potential well. As a warm-up for the hydrogen atom, let us extend our discussion of infinite potential wells to two and three dimensions.

### **Rectangular Corral**

Figure 39-13 shows the rectangular area to which an electron can be confined by the two-dimensional version of Fig. 39-2 - a two-dimensional infinite potential well of widths  $L_x$  and  $L_y$  that forms a rectangular corral. The corral might be on the surface of a body that somehow prevents the electron from moving parallel to the z axis and thus from leaving the surface. You have to imagine infinite potential energy functions (like U(x) in Fig. 39-2) along each side of the corral, keeping the electron within the corral.

Solution of Schrödinger's equation for the rectangular corral of Fig. 39-13 shows that, for the electron to be trapped, its matter wave must fit into each of the two widths separately, just as the matter wave of a trapped electron must fit into a one-dimensional infinite well. This means the wave is separately quantized in width  $L_x$  and in width  $L_y$ . Let  $n_x$  be the quantum number for which the matter wave fits into width  $L_x$ , and let  $n_y$  be the quantum number for which the matter wave fits into width  $L_y$ . As with a one-dimensional potential well, these quantum numbers can be only positive integers. We can extend Eqs. 39-10 and 39-17 to write the normalized wave function as

$$\psi_{nx,ny} = \sqrt{\frac{2}{L_x}} \sin\left(\frac{n_x \pi}{L} x\right) \sqrt{\frac{2}{L_y}} \sin\left(\frac{n_y \pi}{L} y\right), \qquad (39-19)$$

The energy of the electron depends on both quantum numbers and is the sum of the energy the electron would have if it were confined along the x axis alone and the energy it would have if it were confined along the y axis alone. From Eq. 39-4, we can write this sum as

$$E_{nx,ny} = \left(\frac{h^2}{8mL_x^2}\right) n_x^2 + \left(\frac{h^2}{8mL_y^2}\right) n_y^2 = \frac{h^2}{8m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right).$$
 (39-20)

Excitation of the electron by photon absorption and de-excitation of the electron by photon emission have the same requirements as for one-dimensional traps. Now, however, two quantum numbers  $(n_x \text{ and } n_y)$  are involved. Because of that, different states might have the same energy; such states and their energy levels are said to be *degenerate*.

#### **Rectangular Box**

An electron can also be trapped in a three-dimensional infinite potential well - a *box*. If the box is rectangular as in Fig. 39-14, then Schrödinger's equation shows us that we can write the energy of the electron as

$$E_{nx,ny,nz} = \frac{h^2}{8m} \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right).$$
(39-21)

Here  $n_z$  is a third quantum number, for fitting the matter wave into width  $L_z$ .

## Checkpoint 4

In the notation of Eq. 39-20, is  $E_{0,0}$ ,  $E_{1,0}$ ,  $E_{0,1}$ , or  $E_{1,1}$  the ground-state energy of an electron in a (two-dimensional) rectangular corral?

established by the central core, thus preserving the central feature of the independent-particle model. These outside nucleons also interact with the core, deforming it and setting up "tidal wave" motions of rotation or vibration within it. These collective motions of the core preserve the central feature of the collective model. Such a model of nuclear structure thus succeeds in combining the seemingly irreconcilable points of view of the collective and independent-particle models. It has been remarkably successful in explaining observed nuclear properties.

#### Sample Problem 42.09 Lifetime of a compound nucleus made by neutron capture

Consider the neutron capture reaction

$${}^{109}\text{Ag} + n \rightarrow {}^{110}\text{Ag} \rightarrow {}^{110}\text{Ag} + \gamma, \qquad (42-35)$$

in which a compound nucleus (<sup>110</sup>Ag) is formed. Figure 42-15 shows the relative rate at which such events take place, plotted against the energy of the incoming neutron. Find the mean lifetime of this compound nucleus by using the uncertainty principle in the form

$$\Delta E \cdot \Delta t \approx \hbar. \tag{42-36}$$

Here  $\Delta E$  is a measure of the uncertainty with which the energy of a state can be defined. The quantity  $\Delta t$  is a measure of the time available to measure this energy. In fact, here  $\Delta t$  is just  $t_{avg}$ , the average life of the compound nucleus before it decays to its ground state.

**Reasoning:** We see that the relative reaction rate peaks sharply at a neutron energy of about 5.2 eV. This suggests that we are dealing with a single excited energy level of the compound nucleus <sup>110</sup>Ag. When the available energy (of the incoming neutron) just matches the energy of this level above the <sup>110</sup>Ag ground state, we have "resonance" and the reaction of Eq. 42-35 really "goes."

However, the resonance peak is not infinitely sharp but has an approximate half-width ( $\Delta E$  in the figure) of about 0.20 eV. We can account for this resonance-peak width by saying that the excited level is not sharply defined in energy but has an energy uncertainty  $\Delta E$  of about 0.20 eV.



**Figure 42-15** A plot of the relative number of reaction events of the type described by Eq. 42-35 as a function of the energy of the incident neutron. The half-width  $\Delta E$  of the resonance peak is about 0.20 eV.

**Calculation:** Substituting that uncertainty of 0.20 eV into Eq. 42-36 gives us

$$\Delta t = t_{\text{avg}} \approx \frac{\hbar}{\Delta E} \approx \frac{(4.14 \times 10^{-15} \text{ eV} \cdot \text{s})/2\pi}{0.20 \text{ eV}}$$
$$\approx 3 \times 10^{-15} \text{ s.} \qquad \text{(Answer)}$$

This is several hundred times greater than the time a 5.2 eV neutron takes to cross the diameter of a <sup>109</sup>Ag nucleus. Therefore, the neutron is spending this time of  $3 \times 10^{-15}$  s *as part of* the nucleus.

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## Review & Summary

**The Nuclides** Approximately 2000 **nuclides** are known to exist. Each is characterized by an **atomic number** *Z* (the number of protons), a **neutron number** *N*, and a **mass number** *A* (the total number of **nucleons** – protons and neutrons). Thus, A = Z + N. Nuclides with the same atomic number but different neutron numbers are **isotopes** of one another. Nuclei have a mean radius *r* given by

$$r = r_0 A^{1/3},$$
 (42-3)

where  $r_0 \approx 1.2$  fm.

Mass and Binding Energy Atomic masses are often reported

in terms of *mass excess* 

$$\Delta = M - A \quad (\text{mass excess}), \tag{42-6}$$

where M is the actual mass of an atom in atomic mass units and A is the mass number for that atom's nucleus. The **binding energy** of a nucleus is the difference

$$\Delta E_{\rm be} = \Sigma (mc^2) - Mc^2 \quad \text{(binding energy)}, \tag{42-7}$$

where  $\Sigma(mc^2)$  is the total mass energy of the *individual* protons and neutrons. The **binding energy per nucleon** is

## A P P E N D I X

# SOME ASTRONOMICAL DATA

#### Some Distances from Earth

To the Moon*	$3.82 \times 10^8 \mathrm{m}$	To the center of our galaxy	$2.2 \times 10^{20} \text{ m}$
To the Sun*	$1.50 \times 10^{11} \mathrm{m}$	To the Andromeda Galaxy	$2.1 \times 10^{22} \text{ m}$
To the nearest star (Proxima Centauri)	$4.04 \times 10^{16} \mathrm{m}$	To the edge of the observable universe	$\sim 10^{26}  {\rm m}$

С

#### \*Mean distance.

#### The Sun, Earth, and the Moon

Property	Unit	Sun	Earth	Moon	
Mass	kg	$1.99 \times 10^{30}$	$5.98 \times 10^{24}$	$7.36 \times 10^{22}$	
Mean radius	m	$6.96 \times 10^{8}$	$6.37 \times 10^{6}$	$1.74 \times 10^6$	
Mean density	kg/m <sup>3</sup>	1410	5520	3340	
Free-fall acceleration at the surface	m/s <sup>2</sup>	274	9.81	1.67	
Escape velocity	km/s	618	11.2	2.38	
Period of rotation <sup><i>a</i></sup>	_	37 d at poles <sup>b</sup> 26 d at equator <sup>b</sup>	23 h 56 min	27.3 d	
Radiation power <sup>c</sup>	W	$3.90 \times 10^{26}$			

<sup>*a*</sup>Measured with respect to the distant stars.

<sup>b</sup>The Sun, a ball of gas, does not rotate as a rigid body.

<sup>c</sup>Just outside Earth's atmosphere solar energy is received, assuming normal incidence, at the rate of 1340 W/m<sup>2</sup>.

Some Properties of the Planets

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto <sup>d</sup>
Mean distance from Sun, 10 <sup>6</sup> km	57.9	108	150	228	778	1430	2870	4500	5900
Period of revolution, y	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Period of rotation, <sup>a</sup> d	58.7	$-243^{b}$	0.997	1.03	0.409	0.426	$-0.451^{b}$	0.658	6.39
Orbital speed, km/s	47.9	35.0	29.8	24.1	13.1	9.64	6.81	5.43	4.74
Inclination of axis to orbit	<28°	≈3°	23.4°	25.0°	3.08°	26.7°	97.9°	29.6°	57.5°
Inclination of orbit to Earth's orbit	7.00°	3.39°		1.85°	1.30°	2.49°	0.77°	1.77°	17.2°
Eccentricity of orbit	0.206	0.0068	0.0167	0.0934	0.0485	0.0556	0.0472	0.0086	0.250
Equatorial diameter, km	4880	12 100	12 800	6790	143 000	120 000	51 800	49 500	2300
Mass (Earth $= 1$ )	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.002
Density (water = 1)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	2.03
Surface value of $g$ , $^{c}$ m/s <sup>2</sup>	3.78	8.60	9.78	3.72	22.9	9.05	7.77	11.0	0.5
Escape velocity, <sup>c</sup> km/s	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	1.3
Known satellites	0	0	1	2	67 + ring	62 + rings	27 + rings	13 + rings	4

<sup>a</sup>Measured with respect to the distant stars.

 ${}^{b}\mathrm{Venus}$  and Uranus rotate opposite their orbital motion.

<sup>*c*</sup>Gravitational acceleration measured at the planet's equator.

<sup>d</sup>Pluto is now classified as a dwarf planet.